REACTIVE IMPULSE AT HIGH IMPACT VELOCITIES

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When a gas moving at high velocity v impacts against a wall, its temperature rises sharply and radiation begins to play an important part in the energy transfer process [1]. After compression by the shock wave the bulk of the gas begins to undergo radiative cooling, which reduces the reactive impulse transmitted to the obstacle. This effect may occur, for example, when low-density meteoric particles strike a solid wall.

In view of the complexity of the phenomenon, we shall consider the special case where the impacting mass is a plane layer of matter of thickness h and density ρ_0. We shall start by assuming that the impact is against a rigid obstacle in a vacuum.

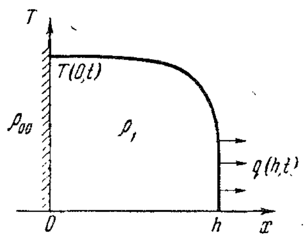

Fig. 1.

Using the laws of conservation at a strong shock front, we find that during time t_1 the layer of gas is compressed to a thickness h_1, while its density ρ_1, pressure p_1, and temperature T_1 are given by the formulas:

$$\rho_1 = \frac{\gamma+1}{\gamma-1}\rho_0, \qquad p_1 = \frac{\gamma+1}{2}\rho_0 v^2, \qquad T_1 = \frac{v^2}{2c_v}, \qquad (1)$$
$$t_1 = \frac{2h}{(\gamma+1)v}, \qquad h_1 = \frac{\gamma-1}{\gamma+1}h, \qquad p = \rho RT, \qquad \left(\gamma = \frac{c_p}{c_v}\right).$$

We shall assume that the specific heat of unit mass of the gas c_v and the ratio of specific heats are constant.

From (1) it follows that in time t_1 the gas transmits to the wall an impulse $p_1 t_1$ equal to the initial momentum $I_0 = \rho_0 hv$. After compression by the shock wave, the layer of gas begins to expand into the vacuum. The corresponding reactive impulse $I_1 = I - I_0$ (here I is the total impulse transmitted to the wall), disregarding radiation, can be computed starting from the exact solution of the equations of gas dynamics [2]:

$$I_1 = \xi(\gamma) I_0 (\xi(1) = 0.796, \ \xi(1.4) = 0.825, \ \xi(3) = 0.865) .$$

The coefficient ξ characterizes the elasticity of the impact and its value, close to unity*, shows that expansion usually occurs under conditions closer to elastic than inelastic impact. The radiation sharply reduces ξ, i.e., inelastic conditions are approached.

The rigorous computation of the energy flux from the heated layer of gas requires the solution of the kinetic equation. However, in estimating the effect of radiation on the reactive impulse, it is possible to confine oneself to a consideration of the extreme case where the path length of the quanta is small $l(\rho, T_1) \ll h_1$. This assumption is admissible, since it turns out that the radiation time is much less than the characteristic time of expansion of the plasma into a vacuum, and the result is relatively independent of the method of de-excitation. We shall write the expression for the reactive impulse in the form:

$$I_1 = \int_{t_1}^{\infty} p(0, t)\, dt = \rho_1 R \int_{t_1}^{t_*} [T(0, t) - T_*]\, dt + \rho_1 R T_* (t_* - t_1) +$$
$$+ \int_{t_*}^{\infty} p(0, t)\, dt \approx \rho_1 R \int_{t_1}^{t_*} [T(0, t) - T_*]\, dt + I_*, \qquad I_* = \xi(\gamma)\rho_0 h \sqrt{2c_v T_*} \qquad (2)$$

where t_* is the time during which it is necessary to take the radiation into account, and T_* is the temperature at which radiation becomes important.

For $l \ll h_1$ the cooling process is described by the diffusion approximation for the kinetic equation, and the heat flux is expressed in terms of the radiation density gradient. For simplicity, we shall assume that the radiation density is from the outset close to the equilibrium value. Initially, even for $l \ll h_1$, the radiation is nonequilibrium owing to the

*Physically, the difference between ξ and unity may be attributed to the redistribution of energy among the gas particles in the nonstationary process.

initial conditions, but it rapidly approaches the equilibrium value and even by the time the free face of the plasma has cooled to $\sim 0.8\ T_1$ the discrepancy is small [3]. At this stage the cooling of the gas is described by the equation of radiative heat transfer:

$$\rho_1 \frac{\partial c_v T}{\partial t} = -\frac{\partial q}{\partial x}, \quad q = -\frac{l}{3}\frac{\partial 4\sigma T^4}{\partial x}, \quad l = b\rho^{-n}T^{\omega}, \qquad \sigma = 5.67 \cdot 10^{-5}\,\frac{\text{erg}}{\text{cm}^2 \cdot \text{sec} \cdot \text{deg}^4}, \tag{3}$$

where l is the radiation path length averaged according to Rosseland and given in the form of an interpolation formula. The boundary conditions for Eq. (3) are:

$$q(0, t) = 0, \qquad q(h_1, t) = 2\sigma T^4(h_1, t) < \sigma T^4(0, t). \tag{4}$$

The last condition follows from the diffusion relation

$$q = 2\sigma T^4 - l\frac{\partial q}{\partial x} \qquad \text{for } l \to 0$$

and expresses the requirement that the kinetic flux at the boundary with the vacuum be equal to half the diffusion flux.

Using the method of moments [5], we can obtain a convenient formula linking the outgoing radiation flux $q(h_1, t)$ and the temperature at the wall $T(0, t)$ (Fig. 1). Equation (3) is equivalent to an infinite number of integral relations obtained by multiplying (3) by x^m ($m = 0, 1, 2, \ldots$) and integrating the expressions obtained with respect to x from 0 to h_1.

We shall satisfy (3) approximately, confining ourselves to the two relations for $m = 0$ and $m = 1$:

$$\rho_1 \frac{d}{dt}\int_0^{h_1} c_v T\, dx = -q(h_1, t), \tag{5}$$

$$\rho_1 \frac{d}{dt}\int_0^{h_1} c_v T x\, dx = -q(h_1, t)\, h_1 + \frac{16\sigma b \rho_1^{-n}}{3k}\left[T^k(0, t) - T^k(h_1, t)\right] (k = \omega + 4). \tag{6}$$

To obtain an approximate solution of Eqs. (4)-(6), we shall use the property of intense heat transfer that makes the temperature distribution close to a "plateau" [6,7]. In this case, in the integrand we can replace T with $T_0(t) \approx T(0, t)$; moreover, bearing in mind that at large k we can neglect the term $T^k(h_1, t)$ compared with $T^k(0, t)$, if $T(h_1, t)$ becomes even slightly less than T_0, we find:

$$q(h_1, t) = B\frac{l_0}{h_1}\sigma T_0^4(t), \qquad B = \frac{32}{3k}, \qquad l_0 = b\rho_1^{-n}T_0^{\omega}. \tag{7}$$

As V. P. Buzdin has shown, it is somewhat more correct to give the temperature distribution in the form:

$$T = T_0(t)\{1 - (x/h_1)^2\}^{1/(k-1)} \tag{8}$$

analogous to that which follows from the solution of [8]. In this case

$$B_1 = \frac{32}{3k\delta}, \qquad \delta = 2 - \frac{\Gamma(1.5 + \alpha)}{(\alpha + 1)\Gamma(\alpha + 1)\Gamma(1.5)}, \qquad \alpha = \frac{1}{k - 1}. \tag{9}$$

From (9) it follows that at large k the value of B_1 is close to that of B.

It is interesting to note that the temperature distribution in the self-similar problem [8] will also be the exact solution of the nonself-similar problem of the propagation of a thermal wave from a source with allowance for radiation of energy from the front [7]. For this problem, which physically is similar to the problem of the cooling of a finite volume of gas, the temperature distribution is given by the solution of [8], which satisfies Eq. (3). This solution must be cut off at a distance $x_1(t)$ determined by the boundary condition expressing the energy balance at the wave front

$$\rho c_v \frac{dx_1}{dt} + \left[\frac{l(T)}{3}\frac{\partial \sigma T^4}{dx}\right]_{x=x_1} = S\left[T(x_1, t)\right]. \tag{10}$$

Since T(x, t) and the law of radiation are known (e. g., $S \sim T^r$, relation (10) serves as an ordinary differential equation for determining $x_1(t)$. In many cases Eq. (10) is solved in quadratures.

In computing the radiation flux from the plasma the motion of the gas was not taken into account. This is correct if q is greater than the adiabatic cooling A for expansion of the gas into a vacuum. Thanks to the intense heat transfer, the Riemann wave can be assumed isothermal [2], and the adiabatic cooling can be computed from the formula:

$$A = \int_0^{x_2(t)} p \frac{\partial V}{\partial t} \rho_1 \, dx_0 = \rho_1 c_1^3, \qquad V = \frac{c_1 t}{\rho_1 x_0}, \qquad p = \rho_1 \frac{x_0}{c_1 t}, \qquad c_1^2 = RT_0, \tag{11}$$

where x_0 is the Lagrangian coordinate of the particle, and $x_2 = c_1 t$ is the width of the Riemann wave.

The characteristic temperature $T_* = T_0(t_*)$, giving the lower limit of applicability of the solution, can be found from the condition q = A. The upper limit of the temperature, at which solution (8) ceases to be correct, can be taken as the temperature T_{**} computed from the equality $l(T_{**}\rho_1) = h_1$. Using (8) and (11), we find

$$T_*^{\omega+2.5} = \frac{\rho_1^{n+1} h_1 R^{1.5}}{\sigma b B}, \qquad T_{**}^{\omega} = \frac{h_1 \rho_1^n}{b}, \; v_* = \sqrt{2c_v T_*}, \qquad v_{**} = \sqrt{2c_v T_{**}}. \tag{12}$$

Figure 2 shows characteristic impact velocities at a rigid wall for an iron meteorite (solid line) and an air jet (broken line). The quantities v_* and v_{**} were calculated for an iron striker of thickness h = 0.1 cm and plotted as a function of the density $y = \rho_0$ g/cm^3. It turns out that in this case the parameters entering into (12) can be assumed approximately equal to $\omega \approx 3$, $h \approx 2$, $\gamma \approx 1.4$, $c_v \approx 8 \times 10^7$ erg/g, $b \approx 8 \times 10^{-23}$ g^2/cm^5 · deg^3.

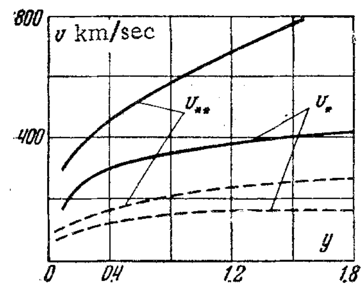

Fig. 2.

The quantities v_* and v_{**} for the air jet were calculated on the assumption that h = 1 cm and are plotted in Fig. 2 as a function of a parameter $y = 2 \times 10^2 \rho_0$ g/cm^3, proportional to the initial density of the jet.

The equation of state of a substance in plasma form is relatively independent of the number of the element. Therefore, Fig. 2 can be used to estimate the critical impact velocity of a number of other substances as well. Iron and air were taken only by of example, since their equations of state have been most thoroughly studied [1, 9].

Using (5) and (8), we get:

$$dt = -\frac{\rho_1^{1+n} c_v h_1^2 dT_0}{\sigma b B T_0^k}.$$

Carrying out the integration in (2), we find the expression for the reactive impulse:

$$I_1 = I_* \left[1 + \frac{C}{\xi(\gamma)\sqrt{2(\gamma-1)}}\right],$$
$$C(\lambda) = \frac{1-\lambda^{\omega+2}}{\omega+2} - \frac{1-\lambda^{\omega+2}}{\omega+3}, \qquad \lambda = \left(\frac{v_*}{v}\right)^2. \tag{13}$$

Since $\omega + 2$ is of the order of 5-6, for a v even slightly in excess of v_* the terms $\lambda^{\omega+2}$ and $\lambda^{\omega+3}$ can be neglected. Therefore from (13) it follows that I_1 remains bounded and practically equal to the reactive impulse for impact at the critical velocity v_*, the ratio I_1/I_0 decreasing as v^{-1}. Formulas (13) are valid if the obstacle is rigid (i.e., the density $\rho_{00} \gg \rho_0$) and does not conduct heat. However, at large impact velocities part of the radiation penetrates the obstacle, dispersal of which increases the limit value I_*.

The boundedness of the reactive impulse is attributable to the fact that the characteristic time of radiation cooling $\tau_1 = \rho_1 h_1^2 c_v / \sigma b B T_1^{k-1} \ll \tau_2 = h_1 / c_1$, where τ_2 is the time of gas-dynamic expansion; therefore in the time the plasma takes to cool to a temperature T_* the wall can not succeed in acquiring much momentum.

Thanks to the strong temperature-dependence of the radiation flux $q \sim T^k$ for $T < T_*$ adiabatic cooling sharply prevails over q. For the purposes of a qualitative estimate of the additionally radiated energy Q for $T < T_*$ it is possible to assume that, as before, q is expressed by (7), but the temperature of the "plateau" $T_0(t)$ is determined by the adiabatic cooling, i.e.,

$$h_1 \frac{dT_0}{dt} = -\frac{(\gamma-1)T_0}{t - t_*} \int_{t_*}^{t} \sqrt{RT_0}\, dt, \qquad Q = \int_{\tau_*}^{t_3} q(T_0)\, dt \qquad \left(t_3 \approx \frac{h_1}{c_*}\right).$$

Carrying out the integration, we find that for large k

$$\frac{Q}{E_*} \approx \frac{1}{4k-1} \qquad (E_* = h_1 \rho_1 c_v T_*) .$$

Therefore Q is only a small part of the energy remaining in the substance after cooling to the temperature T_*, and its effect on the reactive impulse is small.

If the density of the impacting object and the obstacle are of the same order, the wall can not be assumed rigid; therefore the temperature rises less on impact, and the critical impact velocity increases.

Using the laws of conservation for the two waves arising at the point of impact (Fig. 3), we find

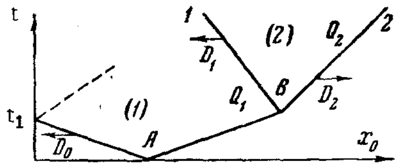

Fig. 3.

$$D_0 = \frac{v(2\varepsilon + 1 - \gamma)}{2(1+\varepsilon)}, \quad u_1 = \frac{v\varepsilon}{1+\varepsilon}, \quad T_{10} = \frac{v^2}{2(1+\varepsilon)c_v}, \qquad (14)$$

$$t_1 = \frac{h}{|\mathbf{v} - \mathbf{D}_0|}, \quad \varepsilon^2 = \frac{\rho_0}{\rho_{00}} .$$

At the moment t_1 the shock wave reaches the boundary with the vacuum and the temperature of the plasma quickly falls to a value $T_{1*} \approx T_*$, which depends relatively little on the thickness of the radiating layer. Therefore, on comparing (1), (12), and (14), we can write the critical impact velocity w_* in the approximate form $w_* \approx \sqrt{1+\varepsilon}\, v_*$.

If the impacting object and the obstacle are made of the same material, then on impact we get the system of waves shown in Fig. 3 (the parameters of the gas in regions 1 and 2 are denoted by the subscripts 1 and 2, respectively). Owing to the intense heat transfer the temperature of the plasma beyond the shock waves is close to the "plateau" and relatively independent of the coordinate, i.e., $T_1 = T_2 \approx T_*$. Therefore the part of the energy Q_1 and Q_2 dissipated at the shock fronts is quickly distributed throughout the gas and partially radiated into the vacuum. The energy balance at shock fronts 2 and 1 may be written thus:

$$\frac{D_2^2}{2} = \frac{(D_2 - u_2)^2}{2} + \frac{\gamma}{\gamma - 1}\frac{p_2}{\rho_2} + Q_2 , \qquad (15)$$

$$\frac{(D_1 - u_1)^2}{2} = \frac{(D_1 - u_2)^2}{2} + Q_1 .$$

Taking into account (15) and using the laws of conservation of mass and momentum at a strong shock front [10], it is easy to find the parameters of the plasma in region 2:

$$u_2 = \sqrt{RT_1(\lambda - 1)}, \quad \rho_2 = \lambda\rho_0, \quad D_2 = \frac{\lambda u_2}{\lambda - 1}, \quad p_2 = \rho_2 R T_1,$$

$$Q_2 = \frac{\lambda - \mu}{1} RT_1, \quad Q_1 = \frac{\lambda + \mu}{\lambda\mu} Q_2, \quad D_1 = \frac{v}{2} - \left(\frac{\lambda R T_1}{\mu}\right)^{1/2}, \qquad (16)$$

$$\mu = \frac{\rho_1}{\rho_0} = \frac{\gamma + 1}{\gamma - 1}, \quad M = \frac{v}{2\sqrt{RT_1}}, \quad \sqrt{\lambda - 1} = M\frac{\lambda - \mu}{\sqrt{\lambda\mu}} .$$

The decay of the shock wave AB into two waves 1 and 2 occurs only if $p_2 > p_1$ and, hence, $\lambda > \mu$. This is fulfilled if the isothermal analogue of the Mach number $M > \sqrt{\lambda - 1}$. If $M < \sqrt{\lambda - 1}$, then $\lambda < \mu$, and energy is supplied to shock front 2, while wave 1 degenerates into an isothermal rarefaction wave.

Using the relations at a strong shock front and for an isothermal Riemann wave [2]

$$u_2 = u_1 - c_1 \ln(\rho_2/\rho_1), \quad u = u_1 + c_1 + x/t ,$$

we find that the gas parameters in region 2 are determined by the same relations (16), in which, however, λ is expressed in terms of M and μ in accordance with the formula:

$$M + \ln \mu/\lambda = \sqrt{\lambda - 1} .$$

Obviously, in this case the radiation from the plasma will be intense only if the energy withdrawn from the two shock waves is greater than the adiabatic cooling at the boundary with the vacuum, i.e.,

$$\rho_1(u_1 - D_1)Q_1 + \rho_0 D_2 Q_2 > \rho_1 c_1^3 \quad \text{or} \quad \frac{\lambda + \mu}{\sqrt{\lambda\mu}} + \frac{\lambda}{\sqrt{\lambda - 1}} \geqslant \frac{2\mu}{\lambda - \mu} . \qquad (17)$$

This criterion is at the same time a necessary condition for the steadiness of the process, which may continue to exist until the relaxation wave (broken line in Fig. 3) overtakes the strong shock front. Using (14), (16), and (17), it is easy to determine the velocity w, at which conditions (17) can exist. Thus, for example,

$$\lambda = 8.4, \quad M = 3.06, \quad w \approx 2.73\ v_* \quad \text{for} \quad \gamma = 1.4\ ,$$
$$\lambda = 14.7, \quad M = 4, \quad w \approx 2.53\ v_* \quad \text{for} \quad \gamma = 1.2\ .$$

From the above it follows that when a mass m_0 impacts at high velocity against a compressible obstacle, complex wave interaction occurs, and the problem of computing the reactive impulse becomes a very difficult one. Therefore we shall confine ourselves solely to a qualitative estimate of the phenomenon.

As Stanyukovich [11] has shown, at high impact velocities, if we disregard radiation, the increase in reactive impulse is proportional to the square of the meteorite velocity

$$I_1 \sim \sqrt{Em_1}, \qquad m_1 \sim v^2, \qquad E_0 \sim m_0 v^2 \rightarrow I_1 \sim v^2\ ,$$

where m_1 is the crater mass. If we take radiation into account, then E and m_1 are reduced.

We shall divide the process of impact into two stages, assuming that up to the moment of release of the principal part of the radiation the shock wave is damped in the obstacle in accordance with the law [12, 13]

$$p = p_0 (m_0 / m)^{\gamma}, \qquad p_0 \sim v^2$$

where m is the entire mass of the gas beyond the shock wave, and p is the pressure at the wave front. Up to the moment $m = m_*$, when the gas cools to the temperature T_*, at which radiation is cut off, the pressure at the front falls to $p_* = p_0(m_0/m_*)^{\gamma}$.

Since $p_* \sim T_*$ in order to estimate p_*, we must substitute the characteristic dimension $m_1^{1/3}$ for h_1 in the radiation cutoff condition $q = A$ [formula (12)]. From (12) we find $p_* \sim T_* \sim m_*^{z}$, $z = {}^1/_3(\omega + 2.5)$ and, since $p_0 \sim v^2$, $m_* \sim v^{2/(\nu+z)}$.

After the plasma has cooled to the temperature T_*, damping of the wave proceeds in accordance with another law and the exponent ν approaches unity [13], i.e., $p \approx p_* m_* / m$ for $m > m_*$.

The mass of the material of the obstacle m_1 contributing to the reactive impulse is determined from the condition that at the wave front p reaches a critical value p_3 which depends on the properties of the material; therefore $m_1 \sim p_* m_* / p_3$. The energy E_1 remaining in the material after the release of radiation is also proportional to $\sim p_* m_*$; therefore

$$I_1 \sim \sqrt{E_1 m_1} \sim p_* m_* \sim v^{\varkappa}, \qquad \varkappa = 2(1 + z) / (\nu + z)\ . \tag{18}$$

In the first stage of impact, while it is possible to neglect adiabatic cooling, the process of energy transfer to the material of the obstacle proceeds in accordance with a law close to that of inelastic impact. In this case all the gas flies in the direction of impact, and its momentum is equal to the initial momentum; then $\nu = 2$ [12].

From (18) it follows that at an impact velocity $v > w_*$ greater than the critical, the reactive impulse increases approximately as $\sqrt{E_0} \sim v$, i.e., considerably more slowly than for the dispersal of a vaporized meteorite and part of the obstacle without radiation.

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